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LETTER TO THE EDITOR

A note on the gravitational field of a rotating radiating source

P A GOODINSON

Department of Mathematics, Queen Elizabeth College, Campden Hill Road, London W8, UK

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Abstract. We present here a generalization of an already known solution.

Kramer (1972) has recently given an exact axially-symmetric nonstationary solution of Einstein's field equations with the energy momentum tensor of a radiation field. In this note we give a generalization of the Kramer solution.

The general metric admitting a null shear free vector field K^α is given by (Robinson and Robinson 1969)

$$\mathrm{d}s^2 = 2P^2\mathrm{d}\zeta\mathrm{d}\bar{\zeta}+2(\mathrm{d}\rho+Z\mathrm{d}\zeta+\bar{Z}\mathrm{d}\bar{\zeta})\mathrm{d}\Sigma+2S(\mathrm{d}\Sigma)^2$$

$$\mathrm{d}\Sigma = a(b\mathrm{d}\zeta+\bar{b}\mathrm{d}\bar{\zeta}+\mathrm{d}\sigma) = K_\alpha \mathrm{d}x^\alpha$$

with $P, Z, \bar{Z}, S$ functions of all four coordinates $(\zeta, \bar{\zeta}, \sigma, \rho)$ and $a, b, \bar{b}$ independent of ρ. Writing $\zeta = (x+\mathrm{i}y)/\sqrt{2}$ the Ricci tensor will be of the form

$$R_{\alpha\beta} = -qK_\alpha K_\beta$$

if

$$a = 1$$

$$b = \mathrm{i}\alpha(1+cg)^{1/2}/(\dot{g}\cosh^2 x), \qquad P^2 = (\rho^2+\Omega^2)/(\dot{g}\cosh x)^2$$

$$Z = \sqrt{(2)}\alpha^2 c\dot{g}\,\mathrm{sech}^2 x\tanh x + \mathrm{i}\alpha\,\mathrm{sech}^2 x\left\{\dot{g}(1+cg)^{1/2}-\rho\left(\frac{(1+cg)^{1/2}}{\dot{g}}\right)^{\cdot}\right\}$$

$$S = -\rho\frac{\ddot{g}}{\dot{g}}-\frac{1}{2}\dot{g}^2+\frac{\alpha^2c^2\dot{g}^2}{4(1+cg)\cosh^2 x}+\frac{\rho m_0\dot{g}^3}{\alpha^2(1+cg)^{3/2}(\rho^2+\Omega^2)}$$

where α, m_0 are constants, g is a function of σ only, $\dot{g}$ denotes $\mathrm{d}g/\mathrm{d}\sigma$ with $\Omega = -\sqrt{(2)}\alpha\dot{g}(1+cg)^{1/2}\tanh x$. The above expressions satisfy the Robinson main and subsidiary conditions.

The Kramer solution is obtained by putting $g = \sigma$, $c \neq 0$. The Kerr stationary vacuum metric (cf Murenbeeld and Trollope 1970) is obtained by putting $g = \sigma$, $c = 0$ and making the transformations

$$\rho = r, \qquad \mathrm{sech}\, x = \sin\theta, \qquad y = -\phi, \qquad \sigma = u$$

with $2\alpha^2 = a^2$ and $2m_0 = ma^2$.

References

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Murenbeeld M and Trollope J R 1970 *Phys. Rev.* D 1 3220–3
Robinson I and Robinson J R 1969 *Int. J. theor. Phys.* 2 231–42